1. Identification of the proposed change

1.1. Title

Library Functions – Hyperbolic Trigonometry

1.2. MDC Proposer and Sponsor

This proposal originates from Ed de Moel.

Motions regarding the status of this document will be made by Taskgroup 5 (Mathematics) of Subcommittee 13 (Data Management and Manipulation).

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1.3. Motion

None.

1.4. History of MDC actions

Date	Document	Action
February 1995	X11/95-13	Final write-up.
January 1995	X11/SC13/94-39	Typographical errors corrected. Presented for promotion to MDC Type A. Approved 21:3:3.
June 1994	X11/SC13/94-28	No changes other than administrative. Presented for promotion to MDC Type A. Too many typographical problems encountered. Document not voted on.
February 1994	X11/SC13/94-10	MUMPS representation of some functions improved. Changed all references from $&MATH.function to $&function^MATH. Presented for promotion to SC#13 Type A. Editorial amendment: passed 17:0:5. Amended document accepted as SC#13 Type A 15:1:5
October 1993	X11/SC13/93-59	MUMPS representation of some functions improved. Presented to replace current SC#13 Type B. Passed 7:1:3.
June 1992	X11/SC13/92-8	Proposal elevated to SC#13 Type B. All names changed from $&LIB.function^HYPER to $&MATH.function.
February 1992	no number	Draft library document discussed in taskgroup
June 1991	X11/SC13/91-24	Collection of mathematical functions elevated to Type B of SC#8.

1.5. Dependencies

(Non)numeric values proposal may redefine representation of infinite and indefinite.

2. Justification of Proposed Change

2.1. Needs

Many people have expressed a concern that the language of MUMPS should be extended with at least the basic mathematical functions. Several years ago, a Library Subcommittee was formed, and within this subcommittee, a taskgroup was formed to produce proposals to add mathematical functions to the language. Initially, the plan of this taskgroup was to add only the most needed functions, after comparison of MUMPS with other programming environments, this plan was changed to offer a library that would be at least as complete as the libraries of Fortran and Ada.

2.2. Existing Practice in Area of the Proposed Change

Since no intrinsic functions are available for mathematical functions, Z-extensions, external call extensions, and MUMPS based approximations are currently in use.

3. Description of the proposed change

3.1. General Description of the Proposed Change

The proposed change adds a number of library functions to the MUMPS function library. Although this proposal uses the 'external library call format', the proposer forsees that some or all of these functions may become intrinsic functions in the future. This proposal, however, deals only with the functionality, not with the method of implementation.

3.2. Annotated Examples of Use

Function calls like

```
   SET X=$%SINH^MATH(ANGLE)
   SET X=$%COSH^MATH(ANGLE)
   SET X=$%TANH^MATH(ANGLE)
   SET X=$%COTH^MATH(ANGLE)
SET PHI=$%ARCSINH^MATH(VALUE)
SET PHI=$%ARCCOSH^MATH(VALUE)
SET PHI=$%ARCTANH^MATH(VALUE)
SET PHI=$%ARCCOTH^MATH(VALUE)
```

would become available.

3.3. Formalization

In the MUMPS Library Specification, add the definition of the library functions below (at the time that this document is written, no document number or section numbering is known for the MUMPS Library Specification). As soon as reasonably possible after establishment of a MUMPS Library Specification, the document editor is directed to update this document to conform with that specification. This proposal does not intend to impinge upon the default package name space.

3.3.1. ARCCOSH Function

3.3.1.1. Library Element Description

ARCCOSH, inverse hyperbolic cosine function.

3.3.1.2. Definition

`ARCCOSH^MATH:REAL(X:REAL,PREC:INTEGER:O)`

The function returns the value of the hyperbolic arccosine in radians of X. The number of significant digits in the hyperbolic arccosine is specified by the optional parameter PREC. If not specified, a default value of 11 digits is assumed for PREC.

3.3.1.3. Domain

X ≥ 1. When the value of parameter X is out of range, an error will result with ecode M28.

3.3.1.4. Range

$%ARCCOSH^MATH(X) ≥ 0.

3.3.1.5. Side effects

None.

3.3.1.6. Example of MUMPS code to implement

```
ARCCOSH(X,PREC) ;
    If X<1 Set $Ecode=",M28,"
    New SQ
    Set SQ=$%SQRT^MATH(X*X-1,PREC)
    Quit $%LOG^MATH(X+SQ,PREC)
```

3.3.1.7. Note(s)

The code in the previous section is an example of a possible implementation of this library function. Vendors are encouraged to provide implementations that offer a better efficiency as well as a greater accuracy.

3.3.2. ARCCOTH Function

3.3.2.1. Library Element Description

ARCCOTH, inverse hyperbolic cotangent function.

3.3.2.2. Definition

`ARCCOTH^MATH:REAL(X:REAL,PREC:INTEGER:O)`

The function returns the value of the hyperbolic arccotangent in radians of X. The number of significant digits in the hyperbolic arccotangent is specified by the optional parameter PREC. If not specified, a default value of 11 digits is assumed for PREC.

3.3.2.3. Domain

X < -1 or X > 1. When the value of parameter X is out of range, an error will result with ecode M28.

3.3.2.4. Range

$%ARCCOTH^MATH(X) < 0 when X ≤ −1 and
$%ARCCOTH^MATH(X) > 0 when X ≥ 1.

3.3.2.5. Side effects

None.

3.3.2.6. Example of MUMPS code to implement

```
ARCCOTH(X,PREC) ;
```

```
    New L1,L2
    Set L1=$%LOG^MATH(X+1,PREC)
    Set L2=$%LOG^MATH(X-1,PREC)
    Quit L1-L2/2
```

3.3.2.7. Note(s)

The code in the previous section is an example of a possible implementation of this library function. Vendors are encouraged to provide implementations that offer a better efficiency as well as a greater accuracy.

3.3.3. ARCSINH Function

3.3.3.1. Library Element Description

ARCSINH, inverse hyperbolic sine function.

3.3.3.2. Definition

`ARCSINH^MATH:REAL(X:REAL,PREC:INTEGER:O)`

The function returns the value of the hyperbolic arcsine in radians of X. The number of significant digits in the hyperbolic arcsine is specified by the optional parameter PREC. If not specified, a default value of 11 digits is assumed for PREC.

3.3.3.3. Domain

Standard.

3.3.3.4. Range

Standard.

3.3.3.5. Side effects

None.

3.3.3.6. Example of MUMPS code to implement

```
ARCSINH(X,PREC) ;
    If X<1 Set $Ecode=",M28,"
    New SQ
    Set SQ=$%SQRT^MATH(X*X+1,PREC)
    Quit $%LOG^MATH(X+SQ,PREC)
```

3.3.3.7. Note(s)

The code in the previous section is an example of a possible implementation of this library function. Vendors are encouraged to provide implementations that offer a better efficiency as well as a greater accuracy.

3.3.4. ARCTANH Function

3.3.4.1. Library Element Description

ARCTANH, inverse hyperbolic tangent function.

3.3.4.2. Definition

`ARCTANH^MATH:REAL(X:REAL,PREC:INTEGER:O)`

The function returns the value of the hyperbolic arctangent in radians of X. The number of significant digits in the hyperbolic arctangent is specified by the optional parameter PREC. If not specified, a default value of 11 digits is assumed for PREC.

3.3.4.3. Domain

$-1 < X < 1$. When the value of parameter X is out of range, an error will result with <u>ecode</u>

M28.

3.3.4.4. Range
Standard.

3.3.4.5. Side effects
None.

3.3.4.6. Example of MUMPS code to implement
```
ARCTANH(X,PREC) ;
    If X<-1 Set $Ecode=",M28,"
    If X>1 Set $Ecode=",M28,"
    Quit $%LOG^MATH(1+X/(1-X),PREC)/2
```

3.3.4.7. Note(s)
The code in the previous section is an example of a possible implementation of this library function. Vendors are encouraged to provide implementations that offer a better efficiency as well as a greater accuracy.

3.3.5. COSH Function
3.3.5.1. Library Element Description
COSH, hyperbolic cosine function.

3.3.5.2. Definition
`COSH^MATH:REAL(X:REAL,PREC:INTEGER:O)`
The function returns the value of the hyperbolic cosine of the angle X in radians. The number of significant digits in the cosine is specified by the optional parameter PREC. If not specified, a default value of 11 digits is assumed for PREC.

3.3.5.3. Domain
Standard.

3.3.5.4. Range
$%COSH^MATH(X) ≥ 1.

3.3.5.5. Side effects
None.

3.3.5.6. Example of MUMPS code to implement
```
COSH(X) ;
    Quit $%EXP^MATH(X)+$%EXP^MATH(-X)/2

COSH(X,PREC) ;
    New F,I,P,R,T,XX
    Set PREC=$Get(PREC,11)+1
    Set @("E=1E-"_PREC)
    Set XX=X*X,F=1,(P,R,T)=1,I=1
    For  Set T=T*XX,F=I+1*I*F,R=T/F+R,P=P-R/R,I=I+2 If -E<P,P<E Quit
    Quit R
```

3.3.5.7. Note(s)
The code in the previous section is an example of a possible implementation of this library function. Vendors are encouraged to provide implementations that offer a better efficiency

as well as a greater accuracy.

3.3.6. COTH Function

3.3.6.1. Library Element Description

COTH, hyperbolic cotangent function.

3.3.6.2. Definition

`COTH^MATH:REAL(X:REAL,PREC:INTEGER:O)`

The function returns the value of the hyperbolic cotangent of the angle X in radians. The number of significant digits in the hyperbolic cotangent is specified by the optional parameter PREC. If not specified, a default value of 11 digits is assumed for PREC.

3.3.6.3. Domain

Standard.

3.3.6.4. Range

$%COTH^MATH(X) < –1 when X < 0 and
$%COTH^MATH(X) > 1 when X > 0.

3.3.6.5. Side effects

None.

3.3.6.6. Example of MUMPS code to implement

```
COTH(X,PREC) ;
   New SINH
   If 'X Quit "INFINITE"
   Set SINH=$%SINH^MATH(X,PREC)
   If 'SINH Quit "INFINITE"
   Quit $%COSH^MATH(X,PREC)/SINH
```

3.3.6.7. Note(s)

The code in the previous section is an example of a possible implementation of this library function. Vendors are encouraged to provide implementations that offer a better efficiency as well as a greater accuracy.

3.3.7. CSCH Function

3.3.7.1. Library Element Description

CSCH, hyperbolic cosecant function.

3.3.7.2. Definition

`CSCH^MATH:REAL(X:REAL,PREC:INTEGER:O)`

The function returns the value of the hyperbolic cosecant of the angle X in radians. The number of significant digits in the hyperbolic cosecant is specified by the optional parameter PREC. If not specified, a default value of 11 digits is assumed for PREC.

3.3.7.3. Domain

Standard.

3.3.7.4. Range

Standard.

3.3.7.5. Side effects

None.

3.3.7.6. Example of MUMPS code to implement

```
CSCH(X,PREC) Quit 1/$%SINH^MATH(X,PREC)
```

3.3.7.7. Note(s)

The code in the previous section is an example of a possible implementation of this library function. Vendors are encouraged to provide implementations that offer a better efficiency as well as a greater accuracy.

3.3.8. SECH Function

3.3.8.1. Library Element Description

SECH, hyperbolic secant function.

3.3.8.2. Definition

`SECH^MATH:REAL(X:REAL,PREC:INTEGER:O)`

The function returns the value of the hyperbolic secant of the angle X in radians. The number of significant digits in the hyperbolic secant is specified by the optional parameter PREC. If not specified, a default value of 11 digits is assumed for PREC.

3.3.8.3. Domain

Standard.

3.3.8.4. Range

0 < $%SECH^MATH(X) ≤ 1.

3.3.8.5. Side effects

None.

3.3.8.6. Example of MUMPS code to implement

```
SECH(X,PREC) Quit 1/$%COSH^MATH(X,PREC)
```

3.3.8.7. Note(s)

The code in the previous section is an example of a possible implementation of this library function. Vendors are encouraged to provide implementations that offer a better efficiency as well as a greater accuracy.

3.3.9. SINH Function

3.3.9.1. Library Element Description

SINH, hyperbolic sine function

3.3.9.2. Definition

`SINH^MATH:REAL(X:REAL,PREC:INTEGER:O)`

The function returns the value of the hyperbolic sine of the angle X in radians. The number of significant digits in the hyperbolic sine is specified by the optional parameter PREC. If not specified, a default value of 11 digits is assumed for PREC.

3.3.9.3. Domain

Standard.

3.3.9.4. Range

Standard

3.3.9.5. Side effects
None.

3.3.9.6. Example of MUMPS code to implement

```
SINH(X) ;
   Quit $%EXP^MATH(X)-$%EXP^MATH(-X)/2

SINH(X,PREC) ;
   New F,I,P,R,T,XX
   Set PREC=$Get(PREC,11)+1
   Set @("E=1E-"_PREC)
   Set XX=X*X,F=1,I=2,(P,R,T)=X
   For  Set T=T*XX,F=I+1*I*F,R=T/F+R,P=P-R/R,I=I+2 If -E<P,P<E Quit
   Quit R
```

3.3.9.7. Note(s)
The code in the previous section is an example of a possible implementation of this library function. Vendors are encouraged to provide implementations that offer a better efficiency as well as a greater accuracy.

3.3.10. TANH Function
3.3.10.1. Library Element Description
TANH, hyperbolic tangent function

3.3.10.2. Definition
TANH^MATH:REAL(X:REAL,PREC:INTEGER:O)
The function returns the value of the hyperbolic tangent of the angle X in radians. The number of significant digits in the hyperbolic tangent is specified by the optional parameter PREC. If not specified, a default value of 11 digits is assumed for PREC.

3.3.10.3. Domain
Standard.

3.3.10.4. Range
–1 < $%TANH^MATH(X) ≤ 1.

3.3.10.5. Side effects
None.

3.3.10.6. Example of MUMPS code to implement

```
TANH(X,PREC) Quit $%SINH^MATH(X,PREC)/$%COSH^MATH(X,PREC)
```

3.3.10.7. Note(s)
The code in the previous section is an example of a possible implementation of this library function. Vendors are encouraged to provide implementations that offer a better efficiency as well as a greater accuracy.

4. Implementation impacts

4.1. Impact on Existing User Practices and Investments

Minimal: existing user-written approximations will continue to be work.

Users may gain precision and performance by using the library functions.

4.2. Impact on Existing Vendor Practices and Investments

None or small. Some implementors already offer some of the proposed functionality.

4.3. Techniques and Costs for Compliance Verification

For each function, a test-suite will have to be developed to check that the function-value is within the limits derived from mathematically correct value and specified error-range (tolerance).

4.4. Legal considerations

None.

5. Closely related standards activities

5.1. Other X11 Proposals (Type A or Type B) Under Consideration

This proposal is one of a series of proposals by the Mathematical Functions taskgroup. The set comprises: trigonometric functions, hyperbolic functions, complex functions, other functions.

5.2. Other Related Standards Efforts

None.

5.3. Recommendations for Co-ordinating Liaison

None.

6. List of Associated Documents

MUMPS/ANSI standard
ISBN 0-201-03809-0 The Art of Computer Programming, Fundamental Algorithms (Knuth)
ISBN 0-486-61272-4 Handbook of mathematical functions (Abramowitz, Stegun)
ISBN 0-06-461019-5 Dictionary of Mathematics (Borowski, Borwein)

7. Issues, Pros and Cons, and Discussion

7.1. 25 October 1993, Dublin Ireland:

Pro 1: Improves existing document.
Con 1: Examples should be verified and annotated with appropriate caveats.
Con 2: Remove MUMPS code in examples.
Sponsor's note: the examples have been verified by the VA offices in Charleston and

Hines.
Con 2 is hard to resolve, because the document format requires the presence of the MUMPS code.

7.2. 26 February 1994, Houston Texas:

Pro 1: Makes MUMPS more general use
Pro 2: Removed objection by significant Government Agency against the use of MUMPS.
Con 1: Normative.
Con 2: Mandates numeric precision which may not be available on all platforms.
Con 1: seems to be a matter of taste.
Con 2: The code, as provided and tested, will provide the stated precision, as long as the requested precision falls within the portability limits.

7.3. 12 June 1994, Reno Nevada:

In order to bring the document in synchronization with the latest Library Specification document, several editorial changes need to be made:

- Function type should follow function name: `name^MATH:type(params)` instead of `name^MATH(params):type`
- `$&` should be replaced by `$%`
- Section 3.3.x.6 should be renamed to **Example of MUMPS code to be implemented**
- Section 3.3.x.7 should be added to emphasize that the MUMPS code that is presented is merely an example and that vendors are encouraged to offer implementations that provide a better efficiency and accuracy

A number of enhancements was recommended:

- Do not use O or I as variable names (replaced by K and N where appropriate)
- The value of several trigonometric functions may be 'infinite'. In some cases, the range is described as 'standard'. As yet, infinity is not included in the 'standard' values of numbers.
- The return type for function CEXP was omitted.
- In the definition of the function ARCCOSH, the function name was misspelled twice (once as ARCSIN, and once as ARCCOS).
- In the definition of the function ARCCSC, the function name was misspelled (as SIN).
- In a number of function definitions, the optional precision parameter was not specified.
- The definition of the functions LOG and LOG10 still contain some text that should have been removed when the function LN was deleted from the proposal.

Individually, the modifications to be made to the document were deemed editorial, in view of the number of modifications, no vote on the document was taken (postponed until an updated document is presented).